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SEPARATION OF MICROPARTICLES USING ULTRASONIC STANDING WAVE

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Abstract

Acoustic particle separation in millimeter scale channel is becoming a useful technology for microfluidics to control micro sized particles or cells. Particle separation methods have widely used in medical, chemical, biological and life science. In this paper, modern method of particle separation focuses on acoustic method. This technology is based on inducing an ultrasonic standing wave inside a millimeter scale channel. Ultrasonic standing wave is generated across a microfluidic channel by piezoelectric transducer integrated in one of the channel wall. This standing wave enables particles to aggregate its pressure node or anti-node lines due to acoustic radiation force. Acoustic radiation force is a function of the density and compressibility of the particles and fluid. In this study, 10 µm particles will be concentrated in water by using ultrasonic standing wave at 2.2 MHz. The objectives of this study are to obtain the fundamental understanding of particle manipulation in millimetre scale rectangular channel at resonance frequency. And then, two-dimensional theoretical model is developed for particle position calculations. The acoustic field numerical models will be constructed using the finite element methods in COMSOL Multiphysics to estimated number of separation lines and acoustic radiation force. The results of this acoustic based COMSOL simulation approach will be compared with the theoretical results.

Keywords: ultrasonic standing wave, acoustic radiation force, millimeter scale channel, microparticles.

Introduction

In recent decades, precise control of microparticles, biomolecules and cells has become increasingly important inengineering, life sciences and medicine. Following this demand, particle manipulation technologies have been developing actively. Particle trapping, levitation, sorting and separation are the different forms of particle manipulation. Using ultrasonic standing wave is one way of manipulating the particles in microfluidic systems. This ultrasound method is suitable for both biological and non-biological suspended particles. The ultrasonic separation principle is based on the acoustic radiation force acting on microparticles. The acoustic radiation force depends on the ratio of the density and compressibility of the suspended microparticles and the surrounding medium, as well as the frequency and energy density of the imposed ultrasound field. Acoustic radiation forces move the particles to acoustic nodes within the acoustic field. Acoustic forces are utilized to separate particles based on their size and density. Ultrasonic method has many advantages. There are no chemical additives requirement, non-contact and no fouling, continuous operation and no effect on cell viability.

In a basic study, King derived an acoustic radiation pressure equation for incompressible particles in liquid. Yosioka et al. developed King's radiation pressure equation for compressible particles in liquid. They discovered the particles aggregate in the nodes or in the anti-nodes in ultrasonic standing wave. Tolt and Fake demonstrated a separation process based on the acoustic radiation force in a stationary ultrasonic standing wave field. Mandralis and Fake collected submerged polystyrene particles with an ultrasonic standing wave, and then moved the particles by periodically changing the frequency. Hill et al developed a model for the calculation of

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particle paths for suspended particles in a fluid for one-dimensional modelling design of resonator for ultrasonic particle manipulation. Most of the models discussed earlier are based on one-dimensional analysis. The two-dimensional modelling of the separation of suspended particle was reported by Francisco J. Trujillo et.al in 2013 by using finite element analysis.

In the present study, a numerical model of an ultrasonic standing wave-based separator is developed. This numerical model performed two dimensional acoustic analysis using COMSOL Multiphysics software. The pressure amplitudes and acoustic radiation force of the ultrasound wave has been determined, the acoustic force acting on particles can be predicted by solving Gor'kov's equation. All the numerical predictions are compared with the theoretical results.

Background Theory

Ultrasound is sound waves with frequencies higher than the upper audible limit of human hearing. Human can hear sound waves with frequencies between about 20 Hz and 20k Hz. Sound above 20kHz is ultrasound and below 20Hz is infrasound. The characteristics of sound wave are wavelength, amplitude, frequency, velocity and time period.

The number of sound waves produced in unit time is called the frequency of sound waves.

$$f = \frac{1}{T} \tag{1}$$

Thus frequency is the reciprocal of the time period of wave. This means that the frequency is increased with decrease in time and vice versa. Time required to produce one complete wave is called time period or time taken to complete on oscillation is called the time period of the sound wave.[8]

Wavelength is the length between two consecutive peaks. The wavelength of the sound wave in a medium is related to the operating frequency and the sound speed in the medium as follows,

$$\lambda = \frac{c}{f} \tag{2}$$

Where c is the speed of sound in fluid medium and f is the frequency of ultrasonic standing wave.

Ultrasonic Standing Wave

A standing wave is two harmonic waves with equal amplitude, frequency and wavelength that are moving in the opposite direction of each other. Standing wave appears when the medium is confined between two boundaries.

A wave traveling to the right along the x-axis is described by the incident wave equation,

$$y_1 = A\sin(kx - \omega t) \tag{3}$$

An identical wave traveling to the left is described by the reflected wave equation,

$$y_2 = A\sin(kx - \omega t) \tag{4}$$

The standing wave equation is

$$y(x,t) = 2A \sin kx \cos \omega t \qquad (5)$$

This means that the total energy associated with standing wave is twice that of incident or reflected wave. The terms x and t are variables for longitudinal position and time. A is the amplitude of the wave and ω is the angular frequency.

Node and Antinode

A node is a point along a standing wave where the waves amplitude is zero and series of location at equally spaced intervals. Pressure of the wave is zero at node points. The location of the node,

$$x_n = \frac{n\lambda}{2} \ , \quad n=0,1,2,3,\ldots \qquad (6)$$

Anti-node is midway between each pair of nodes and the amplitude is maximum.

$$x_{an} = \left(n + \frac{1}{2}\right)\frac{\lambda}{2} \ , \quad n=0,1,2,3,\ldots \qquad (7)$$

The distance between two nearest node or two nearest antinode is λ/2. The distance between consecutive node and antinode is λ/4.

The number of nodes or separation lines N can be estimated the following formula,

$$N = \frac{W}{\lambda/2} \qquad (8)$$

Two Dimensional Layered Resonator Model

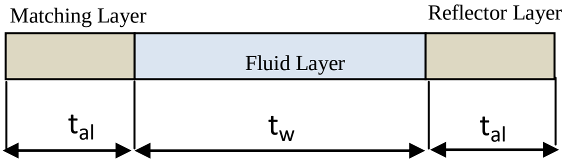

Figure 1 Two-dimensional Layered Resonator Model

Figure 1 shows two dimensional model of layered resonator. There are three layers in the resonator matching, fluid and reflector layers. This resonator is made with aluminium. The piezoelectric transducer and adhesive layer are not considered in this model.

$$t_{al} = n \times \frac{\lambda_{al}}{4} \ , \quad n=1,3,5,7,\ldots \qquad (9)$$

$$t_w = n \times \frac{\lambda_w}{2} \ , \quad n=1,2,3,4,\ldots \qquad (10)$$

Acoustic Radiation Force

King first provided an expression for acoustic radiation force on a small, rigid, incompressible spherical object in an fluid, where the wavelength of the acoustic wave is much larger than the radius of the object. Yosioka and Kawasima expanded King's theory to compressible objects and expressed time-averaged radiation force on a sphere in a plane USW field with acoustic energy density. The acoustic radiation force was calculated using Gor'kov's formulation. The acoustic radiation force is defined as a function of a potential U.

$$F_a = -\nabla U \qquad (11)$$

The force is the negative gradient of the force potential. Where the potential U is defined as,

$$U = 2\pi\rho_w r^3 \left[\frac{f_1}{3\rho_w{}^2 c_w{}^2}\langle p\rangle^2 - \frac{f_2}{2}\langle v\rangle^2\right] \qquad (12)$$

$$f_1 = 1 - \frac{\rho_w c_w{}^2}{\rho_p c_p{}^2} \quad , \quad f_2 = \frac{2\left(\rho_p - \rho_w\right)}{2\rho_p - \rho_w} \qquad (13)$$

So, r is the radius of particles, c is the speed of sound, p is the acoustic pressure and v is the fluid particles velocity. w and p refer to water and particle properties. And the acoustic number k is defined by $2\pi/\lambda$. The notable aspect of the acoustic contrast factor is the possible sign change depending on the densities and compressibility of fluid and particles.

$$\emptyset = \frac{1}{3}\left(\frac{5\,\rho_p - 2\,\rho_w}{2\,\rho_p + \rho_w} - \frac{\rho_w\; c_w^2}{\rho_p\; c_p^2}\right) \qquad (14)$$

Where, ρ_p = densities of particle

ρ_w = densities of water

c_p = sound speed of particle

c_w = sound speed of water

From the equation (14), when ø is positive, particles aggregate at the nodes of the sound pressure profile and other they aggregate at the anti-nodes.

The forces that affect the particle movement are not only acoustic radiation force, but also the drag forces on the particles due to the fluid flow. The stokes drag force F_{drag} can be expressed as:

$$F_{drag} = 6\pi\mu r\left(v_f - v_p\right) \qquad (15)$$

For a spherical particle with radius r, moving at the fluid and particle velocity are v_f and v_p through a liquid with viscosity μ.

Numerical Analysis

This simulation performed with COMSOL Multiphysics software in two dimensional finite element model of fluid layer was constructed using acoustic module. The rectangular model is 6mm width and 50mm length. For this simulation, maximum mesh size of the element is around λ/10. The input parameters for the simulations are density of water $\rho_w = 1000\ \mathrm{kg/m^3}$, viscocity $\mu = 0.9\times10^{-3}$ kg/ms, speed of sound of water c = 1500 m/s, particle diameter = 10µm and density of particle $\rho_p = 1050\ \mathrm{kg/m^3}$.

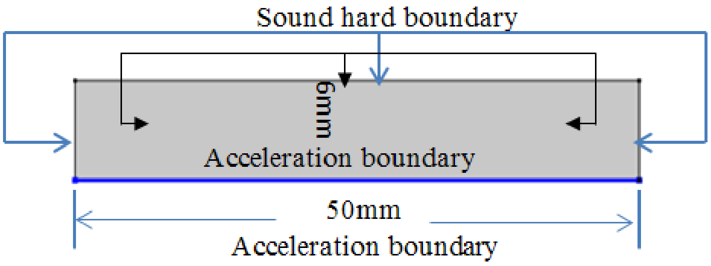

Figure 2 Two Dimensional Channel Geometry

Figure 2 shows the simplified model with boundary conditions. All the rigid walls of the channel were modeled as sound hard boundaries, except the wall where the piezo-actuator was attached. This side of the model was simplified by using an acceleration boundary:

$$-n\left(-\frac{1}{\rho_c}\left(\nabla p_t - q_d\right)\right) = a_n \tag{14}$$

The sound hard boundary was defined by:

$$-n\left(-\frac{1}{\rho_c}\left(\nabla p_t - q_d\right)\right) = 0 \tag{15}$$

Result and Discussion

For the acoustic analysis, a two-dimensional finite element model was constructed using acoustic module in COMSOL. Since the acoustic contrast factor between polystyrene particles and water is approximately 0.78, particles are supposed to be attracted to the pressure nodes. Pressure profiles in the 6 mm width of fluid ch annel can be estimated using finite element model.

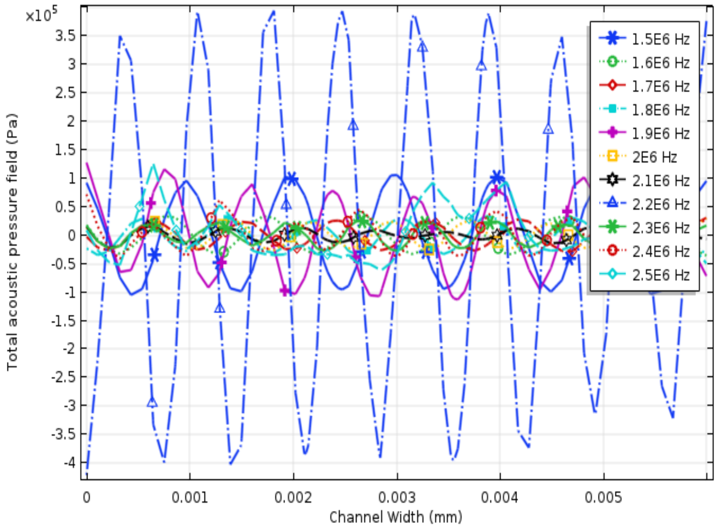

Figure 3 Pressure Profile in Fluid Layer at Frequency 1.5 to 2.5 M Hz

Figure 3 shows the pressure amplitude for frequencies of 1.5 to 2.5 M Hz. The strong changes of the pressure amplitudes at different frequencies are caused by resonance. Operating the system at frequencies close to a resonant frequency will yield higher pressure. The maximum pressure amplitude throughout the process was found at 2.2 MHz.

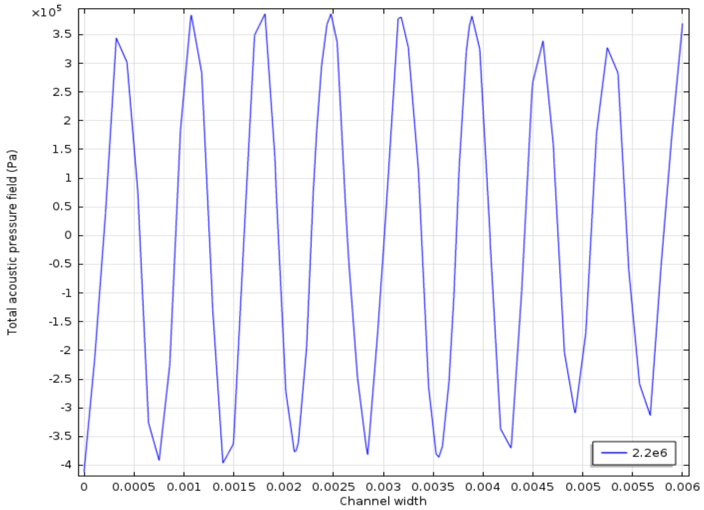

Figure 4 Pressure Profile in Fluid Layer at Frequency 2.2 MHz

As the Figure 4 shows, an ultrasonic standing wave with 8.5λ in the chamber was formed at 2.2 MHz, and the 10µm polystyrene particles aligned on the 17 pressure nodal lines.

Table 1 Theoretical and Simulation of Separation Lines in 6mm Channel

	Theoretical	**Simulation**	**Discrepancy (%)**
λ (mm)	0.6818	0.6729	6
No;of node points	17.6	17	4

As can be seen in the Table I, theoretical and simulation calculated the value of wavelength and node points. The discrepancy of these two is less than 10. So the result is acceptable.

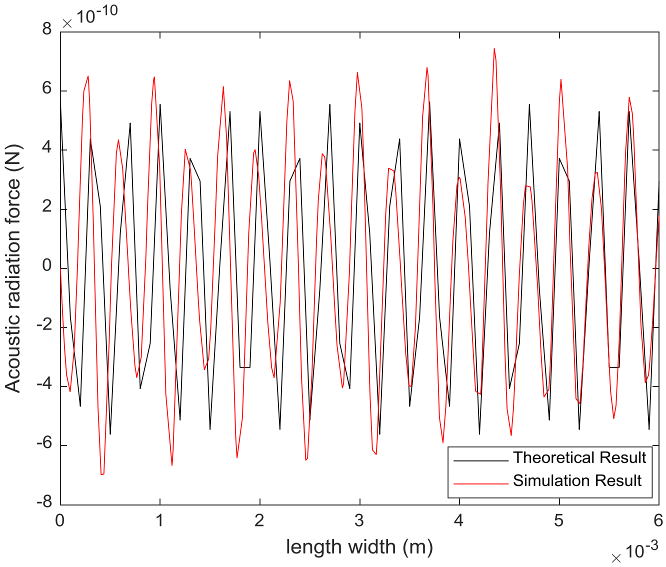

Figure 5 Theoretical and Simulation of Acoustic Radiation Force

Figure 5 shows the theoretical and simulation result of acoustic radiation force for 6mm rectangular channel at frequency 2.2 MHz. As can be seen from the figure, both results are same phase and exactly the same trend.

Conclusion

In this study, basic theory of acoustics in general is first discussed. Next, theory of ultrasonic standing wave is discussed and amount of separation lines in the fluid layer model are estimated. This paper presented theoretical, experimental and numerical model to predict the number of node points and value of wavelength of the ultrasonic standing wave. Theoretical and simulation result of acoustic radiation force are same phase and equal trend in comparison. Two-dimensional acoustic simulation models were built to predict the ultrasound and flow fields inside the fluid layer.

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